Evolution of a Massive Scalar Fields in the Spacetime of a Tense Brane Black Hole

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In the spacetime of a d-dimensional static tense brane black hole we elaborate the mechanism by which massive scalar fields decay. The metric of a six-dimensional black hole pierced by a topological defect is especially interesting. It corresponds to a black hole residing on a tensional 3-brane embedded in a six-dimensional spacetime, and this solution has gained importance due to the planned accelerator experiments. It happened that the intermediate asymptotic behaviour of the fields in question was determined by an oscillatory inverse power-law. We confirm our investigations by numerical calculations for five- and six-dimensional cases. It turned out that the greater the brane tension is, the faster massive scalar fields decay in the considered spacetimes.

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I. INTRODUCTION

In recent years the subject of the late-time behaviour of various fields in the spacetime of a collapsing body has acquired a great deal of attention due to the fact that regardless of details of the collapse or the structure and properties of the collapsing body the resultant black hole may be described by a few parameters such as mass, charge and angular momentum, *black holes have no hair.*

In [1] it has been pointed out that the late-time behavior is dominated by the factor $t^{-(2l+3)}$, for each multipole moment l, while in [2] that the decay-rate along null infinity and along the future event horizon is governed by the power laws $u^{-(l+2)}$ and $v^{-(l+3)}$, where u and v were the outgoing Eddington-Finkelstein (ED) and ingoing ED coordinates. The scalar perturbations in the Reissner-Nordström (RN) spacetime were treated in [3]. It happened that a charged hair decays slower than a neutral one [4]-[6]. Burko [7] reported the late-time tails in gravitational collapse of a self-interacting (SI) fields in the background of Schwarzschild solution. On the other hand, RN solution at intermediate late-time was considered in [8]. The nearly extreme case of the RN spacetime was elaborated analytically in article [9].It was concluded that the inverse power law behavior of the dominant asymptotic tail is of the form $t^{-5/6}\sin(mt)$. It turned out [10] that the oscillatory tail of scalar field in Schwarzschild spacetime has the decay rate of $t^{-5/6}$ at asymptotically late time. The power-law tails in the evolution of a charged massless scalar field around a fixed background of dilaton black hole was studied in [11], while the case of a self-interacting scalar field was elaborated in [12]. The analytical proof of the above mentioned behaviour of massive scalar hair in the background of a dilaton black hole in the theory with arbitrary coupling constant between $U(1)$ gauge field and dilaton field was given in [13]. It was revealed [14] that the asymptotic behaviour of massive Dirac fields in the background of Schwarzschild black hole was independent of the multipole number of the wave mode and of the mass of the Dirac field. Their decay was slower comparing to the decay of a massive scalar field. The behaviour of Dirac fields in the spacetime of RN black hole was treated in [15] while the case of a stationary axisymmetric black hole background was studied numerically in [16]. In [17] both the intermediate late-time tail and the asymptotic behaviour of the charged massive Dirac fields in the background of a Kerr-Newmann black hole was under consideration and it was demonstrated that the intermediate late-time behaviour of the fields in question was dominated by an inverse power-law decaying tail without any oscillation. Massive vector field obeying the Proca equation of motion in the background of Schwarzschild black hole was studied in [18] where it was revealed that at intermediate late times, three functions characterizing the field have different decay law depending on the multipole number l. On the contrary, the late-time behaviour is independent on l, i.e., the late-time decay law is proportional to $t^{-5/6}\sin(mt)$. In [19] the analytical studies concerning the intermediate and late-time decay pattern of massive Dirac hair on the dilaton black hole were conducted. Dilaton black hole constitutes a static spherically symmetric solution of the theory being the low-energy limit of the string theory with arbitrary coupling constant α. In [20] it was assumed that the massive Dirac hair propagated in a static spherically symmetric spacetime with asymptotically flat metric. The intermediate late time behaviour was found numerically and the final decay pattern at very late times was calculated analytically. The metric in question, was characterized by the ADM mass and some other parameters of the background fields. The intermediate and late-time decay of massive scalar hair on static brane black holes was elaborated in [21], while the decay of massive Dirac hair in the spacetime of black hole in question was studied in [22].

For n-dimensional static black holes, the *no-hair* theorem is quite well established [23]. One should also mention about some attempts to establish uniqueness theorem for another kind of black objects, black rings. The uniqueness theorem for five-dimensional stationary the so-called Pomeransky-Sen'kov black ring was found in [24] while black ring σ-model uniqueness theorem was given in [25]. On the other hand, the mechanism of decaying black hole hair in higher dimensional static black hole case concerning the evolution of massless scalar field in the n-dimensional Schwarzshild spacetime was determined in [26]. The late-time tails of massive scalar fields in the spacetime of n-dimensional static charged black hole was elaborated in [27] and it was found that the intermediate asymptotic behaviour of massive scalar field had the form $t^{-(l+n/2-1/2)}$. This pattern of decay was checked numerically for $n = 5, 6$. Quasi-normal modes for massless Dirac field in the background of n-dimensional Schwarzschild black hole were studied in [28].

Due to the fact that forthcoming Large Hadron Collider (LHC) may open the way for producing microscopic black holes which will quickly decay through the Hawking radiation, interest in tensional brane black holes rapidly grows. In [29] the exact metric describing a black hole localized on a codimensional-2 brane was constructed. It turned out that the line element $d^2\Omega_{d-2}$ on the unit $(d-2)$-dimensional sphere is replaced by the metric on a unit sphere but with the wedge removed from the polar coordinates, i.e., one gets some kind of a *topological defect* piercing the black hole in question. The finite brane tension modifies the standard results achieved in the case of a brane black hole with negligible tension.

This discovery bears directly on the question of the underlying physics. Namely, for the first time it was observed in the semi-classical description of the black hole decay process [30]. In [31] the metric of codimensional brane black hole was generalized to the rotating case, while article [32] was devoted to massless fermion excitations on a tensional three-brane embedded in six-dimensional spacetime. Various aspects of the tense brane black hole physics were studied. Among all, quasi-normal modes, bulk scalar emission, grey-body factors and the behaviour of scalar perturbations in the background of a black hole localized on a tensional three-brane in a world with two large extra dimensions were elaborated in [33]-[36]. Having all in mind it will not be amiss to pay more attention to the decay process of massive scalar hair in the background of a tense brane black hole.

In this paper our investigations will be devoted to studies of d-dimensional tensional brane black hole. In the case of $d = 5$, 6 we provide numerical analysis of the intermediate late-time behaviour of massive scalar fields in the black hole backgrounds in question.

The remainder of the paper is as follows. In Sect. II we shall discuss the massive scalar field behaviour in the spacetime of d-dimensional static spherically symmetric tense brane black hole. In Sect. III we treat numerically the problem of five and six-dimensional black hole in question. We conclude our investigations in Sect. IV.

II. MASSIVE SCALAR FIELDS IN A TENSE BRANE BLACK HOLE BACKGROUND

Recent progress in string theory and brane world models led us to an intense interest in higher-dimensional black holes. In model with large extra dimensions black hole size is much smaller than the effective size of the extra dimensions. Simply additional dimensions are considered as having infinite extent. Models with large extra dimensions acquire much more attention also because of an interesting possibility of lowering the fundamental scale of gravity down to order of TeV. It can be argued that future experiments, which involve high-energy particles collisions at future colliders, hold great promise for illuminating the nature of mini black holes. Especially, forthcoming LHC brings the possibility of creating microscopic black holes which will disappear quickly after creation with the emission of the Hawking radiation. Most examinations of extra-dimensions black holes and their physics were devoted to the zero brane tension case. But in principle finite brane tension may modify the physics of a tensional brane black hole. The nonzero tension on the brane can curve the brane as well as the bulk. Therefore more attention should be directed to the studies of the afore mentioned black objects. A tense brane black hole is locally a higher-dimensional Schwarzschild solution [29] threaded by a tensional brane. This caused that a deficit angle appeared in the $(d-2)$-dimensional unit sphere line element. In what follows, our considerations will be devoted to studies of such kind of black objects pierced by a codimension-2 brane.

In our considerations we take into account the following equation of motion for massive scalar field:

$$\nabla^i \nabla_i \tilde{\psi} - m^2 \tilde{\psi} = 0, \tag{1}$$

where m is the mass of the field. One should study its behaviour in the background of static spherically symmetric d-dimensional black hole. The line element of such a black hole is subject to the relation

$$ds^2 = -f^2(r)dt^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{d-2}^2, \tag{2}$$

where $f^2(r) = 1 - \left(r_0/r\right)^{d-3}$, r_0 is the radius of the black hole event horizon, while $d\Omega_{d-2}^2$ is a line element on S^{d-2} sphere provided by the relation

$$d\Omega_{d-2}^2 = d\theta^2 + \sum_{i=2}^{d-3}\prod_{j=1}^{i-1}\sin^2\phi_j d\phi_j. \tag{3}$$

In our considerations we take into account S^{d-2} sphere threaded by a codimension-2 brane, so the range of one of the angles, let us say ϕ_i, will be $0 \le \phi_i \le 2\pi B$.

Consequently, in the case of d-dimensions we resolve the scalar field function as follows:

$$\tilde{\psi} = \sum_{l,n}\frac{1}{r^{\frac{d-2}{2}}}\psi_n^l(t,r)Y_l^n(\theta,\phi), \tag{4}$$

where Y_l^n are scalar spherical harmonics on the unit $(d-2)$-sphere. For the case when $(d-2)$ sphere is pierced by a topological defect, one can introduce *hyper-spherical* coordinates in the form: $0 < r < \infty$, $0 \le \theta \le \pi$, $-\pi \le \phi_1 \le \pi$, ..., $-\pi B \le \phi_{d-3} \le \pi B$. Having in mind results of [37], one may introduce $k = \sum_{i=0}^{d-4} n_i$, where n_i are the integers separation constants and now the multipole number l has the form

$$l = k + \tilde{n}, \tag{5}$$

where $\tilde{n} = \frac{n}{B}$ expresses the existence of a topological defect. Further on, it is convenient to make the change of variables. Namely, let us define the tortoise coordinates y

$$dy = \frac{dr}{\left(1 - \left(\frac{r_0}{r}\right)^{d-3}\right)}. \tag{6}$$

Using Eq.(4) and relation (6) one has the following equations for each multipole moment:

$$\psi_{,tt} - \psi_{,yy} + V\psi = 0, \tag{7}$$

where the form of the strictly radial potential $V(r)$ yields

$$V = f^2(r)\left[\left(\frac{d-2}{2}\right)\frac{1}{r^{\frac{d-2}{2}}}\frac{d}{dr}\left(r^{\frac{d}{2}-2}\,f^2(r)\right) + \frac{\left(k+\mid\tilde{n}\mid\right)\left(k+d-3+\mid\tilde{n}\mid\right)}{r^2} + m^2\right]. \tag{8}$$

The square of the Laplace operator on S^{d-2} threaded by a topological defect is given by

$$\lambda^2 = \left(k+\mid\tilde{n}\mid\right)\left(k+d-3+\mid\tilde{n}\mid\right). \tag{9}$$

One can introduce an auxiliary variable ξ which satisfies $\xi = f(r)\psi$. Next, one assumes that the observer and the initial data are in the region where the following condition is satisfied $r \ll \frac{r_0}{(r_0 m)^2}$ and moreover we consider the intermediate asymptotic behaviour of massive scalar field, i.e., $r \ll t \ll \frac{r_0}{(r_0 m)^2}$. If we consider further, the explicit dependence on time in function ψ of the form $e^{-iomegat}$, where ω is frequency, then we achieve

$$\frac{d^2\xi}{dr^2} + \left[\omega^2 - m^2 - \frac{\nu(\nu+1)}{r^2}\right]\xi = 0, \tag{10}$$

where $\nu = \left(k+\mid\tilde{n}\mid-2+\frac{d}{2}\right)$. Solution of Eq.(10) can be found by virtue of the method used in [38]. It happened that the asymptotic intermediate behaviour of scalar field at fixed radius implies

$$\psi \sim t^{-(k+|\tilde{n}|+\frac{d}{2}-\frac{1}{2})}, \tag{11}$$

while the intermediate behaviour of massive scalar fields at the future black hole horizon H_+ is dominated by an oscillatory power law tails of the form as follows:

$$\psi \sim v^{-(k+|\tilde{n}|+\frac{d}{2}-\frac{1}{2})}\sin(mv), \tag{12}$$

where v is is advanced null coordinate.

In what follows we shall treat the problem of five and six-dimensional static spherically symmetric black hole threaded by a topological defect. The six-dimensional case is of a special interest. In spherical coordinates its line element implies

$$ds^2 = -\left[1 - \left(\frac{r_0}{r}\right)^3\right]dt^2 + \frac{dr^2}{\left[1 - \left(\frac{r_0}{r}\right)^3\right]} +$$
$$+ r^2(d\theta^2 + \sin^2\theta[d\phi^2 + \sin^2\phi(d\chi^2 + B^2\sin^2\chi d^2\psi)]). \tag{13}$$

Now, the parameter $B = 1 - \frac{\lambda}{2\pi M_*}$ measures the deficit angle about an axis which is parallel to the three-brane with finite tension λ. M_* is the fundamental mass scale of six-dimensional gravity. The black hole horizon is at the distance of $r_0 = r_s/B^{1/3}$, where r_s is the six-dimensional Schwarzschild radius given in terms of the ADM mass by the relation

$$r_s = \left(\frac{1}{4\pi^2}\right)^{\frac{1}{3}}\frac{1}{M_*}\left(\frac{M_{BH}}{M_*}\right)^{\frac{1}{3}}. \tag{14}$$

One can remark that the main effect of the brane tension is to change the relation between the black hole mass and the event horizon radius.

In the next section we shall check our predictions numerically confining our attention to the case of $d = 5$ and $d = 6$.

III. NUMERICAL RESULTS

Due to the difficulties in solving differential Eqs. of motion in terms of adequate special functions for arbitrary spacetime dimension, we treat them numerically. By the method presented in [2, 27] we shall find the solution of the equation of motion transformed into retarded and future null coordinates (u, v). Consequently, we achieve the following:

$$4\psi_{,uv} + V\psi = 0. \tag{15}$$

Equation (15) will be solved on an uniformly spaced grid using the explicit difference scheme. As was pointed out previously, the late time evolution of a massive scalar field is independent of the form of the initial data. In order to perform our calculations we start with a Gaussian pulse of the form

$$\psi(u = 0, v) = A\exp\left(-\frac{(v - v_0)^2}{\sigma^2}\right) \tag{16}$$

Because of the linearity of the relation (7) one has freedom in choosing the value of the amplitude A. For our purpose we fix it as $A = 1$. The rest of the initial field profile parameters will be taken as $v_0 = 50$ and $\sigma = 2$.

We begin our numerical studies with the evolution of scalar field ψ on a future timelike infinity i_+. In the considerations we approximate this case by the field at fixed radius $y = 50$. The numerical results for $k = 0$, $n = 1$, $B = 0.9$ and mass of the field $m = 0.01$ for different spacetime dimensions are depicted in Fig. 1. One should remark that the initial evolution of the scalar field in question is determined by prompt contribution and quasi-normal ringing. Then, with the passage of time a definite oscillatory power-law fall-off is manifest. We get the following power-law exponents -3.14 and -3.64 for $d = 5$, 6, respectively. Having in mind Eq.(11) one can remark that we get good agreement between numerical calculations and analytically predicted values: -3.11 and -3.61, accordingly, that is the discrepancies are only 1% and 0.8%. For all considered cases, the discrepancies between analytical and numerical values range from 0.2% up to 1.8%. The period of oscillation is $T = \pi/m \simeq 314.5 \pm 0.5$.

Next, in Fig.2 we elaborate the evolution of the massive scalar field on the black hole future event horizon H_+ approximated by $\psi(u = 10^4, t)$. We studied the evolution as a function of v for different spacetime dimensions, i.e., $d = 5$, 6. The calculations parameters were as follows: $k = 0$, $n = 1$, $B = 0.9$, $m = 0.01$. The power-law exponents and the period of oscillation are nearly the same as in Fig. 1. Then, we studied the behaviour of ψ due to the change of brane tension B. In our calculations we take $B = 0.9$, 0.7, 0.5 (for $k = 0$, $n = 1$), while $d = 5$, 6, respectively. The obtained results are presented in Figs. 3 and 4. The received power-law exponents are -3.14, -3.47, -4.04 for the five-dimensional case and -3.64, -3.97, -4.54 for the six-dimensional spacetime. We observe that the bigger B is the smaller power-law exponents we get, i.e., the greater brane tension is exerted the faster decay of massive scalar hair on the considered black hole spacetimes is observed.

We also studied the power-law decay behaviour for different $n = 0,\ 1,\ 2$ (Fig.5) and established $B = 0.9$ and $k = 0$ in five-dimensional black hole spacetime, where we obtained the power-law exponents equal to $-2.03,\ -3.14,\ -4.26$. It turned out, that the same calculation parameters reveal in six-dimensional case the power-law exponents of the form $-2.53,\ -3.64,\ -4.78$ (see Fig.6). Figs. 7 and 8 depict the dependence of the decay pattern on the multipole number k. The case of different masses of the scalar field in the black hole background was treated in Figs. 9 and 10. Namely, we studied the late-time decay rate on future timelike infinity in five-dimensional black hole spacetime for different masses of the field in question (see Fig. 9). The decay rate was a power-law fall-off with the slope -3.14. The six-dimensional case was presented in Fig. 10. For various masses of scalar field one gets the power-law fall-off with the slope of the curve equal to -3.64.

IV. CONCLUSIONS

Motivated by the forthcoming possibility of producing in LHC experiments microscopic black holes and the necessity of studying their properties we have elaborated the intermediate behaviour of massive scalar fields in the background of a static d-dimensional tensional brane black hole. Due to the tremendous difficulties in solving equations of motion for the massive scalar field in general d-dimensional black hole spacetime, we studied numerically the case of a five and six-dimensional spacetime. The case of a six-dimensional black hole pierced by a topological defect is of a special interest due to the interpretation of this line element as a black hole residing on a tensional 3-brane embedded in a six-dimensional spacetime. We checked analytical calculations by numerical ones and obtained good agreement with analytically predicted values. Numerical studies of the intermediate late-time behaviour of massive scalar field on future timelike infinity and on the future event horizon of tense brane black hole revealed the inverse power-law decay of the field in question. The inverse power-law decay pattern was checked numerically for $d = 5,\ 6$. It turned out that the intermediate late time behaviour depended on the mass of the scalar field, as well as the multipole number of the wave mode and parameters characterizing topological defect. The same situation takes place in the five-dimensional case. As we have mentioned, for a six-dimensional codimension-2 brane black hole its event horizon radius is strictly connected with three-brane tension. Thus, our numerical calculations revealed that the greater brane tension is the faster massive scalar field decays in the spacetime in question.

To finish with let us give some remarks concerning the four-dimensional case. In four-dimensions the considered line element reduce to the case found in [39]. It describes a thin cosmic string passing through a Schwarzschild black hole. It was revealed that it constituted the limit of a much more realistic situation when a black hole was pierced by a Nielsen-Olesen vortex [40]. In the extremal limit of the black hole under consideration a completely new phenomenon occurred. The flux of the vortex is expelled from the black hole (for some range of black hole parameters), rather like the flux is expelled from a superconductor (the so-called *Meissner effect*). It happened that in the case of the extremal black holes in dilaton gravity one has always expulsion of the Higgs fields from their interiors [41].

Using the method presented in [37], it could be easily found that the eigenvalues of the Laplace operator on a S^2 unit sphere pierced by a cosmic string, i.e., $\lambda^2 = (k + \mid \tilde{n} \mid)(k + \mid \tilde{n} \mid + 1)$, where $k = 0, 1, 2 \ldots$ It turned out that the crucial role is played by the factor B connected with a mass per unit length of a string, because of the fact that $\tilde{n} = n/B$. Having in mind [19, 20] it can be shown that the intermediate asymptotic behaviour of a scalar field depends on the field's parameter mass, cosmic string parameter as well as the multipole number k. The late-time behaviour reveals that the power law decay rate which is independent of the parameters in question has the form of $t^{-5/6}$.

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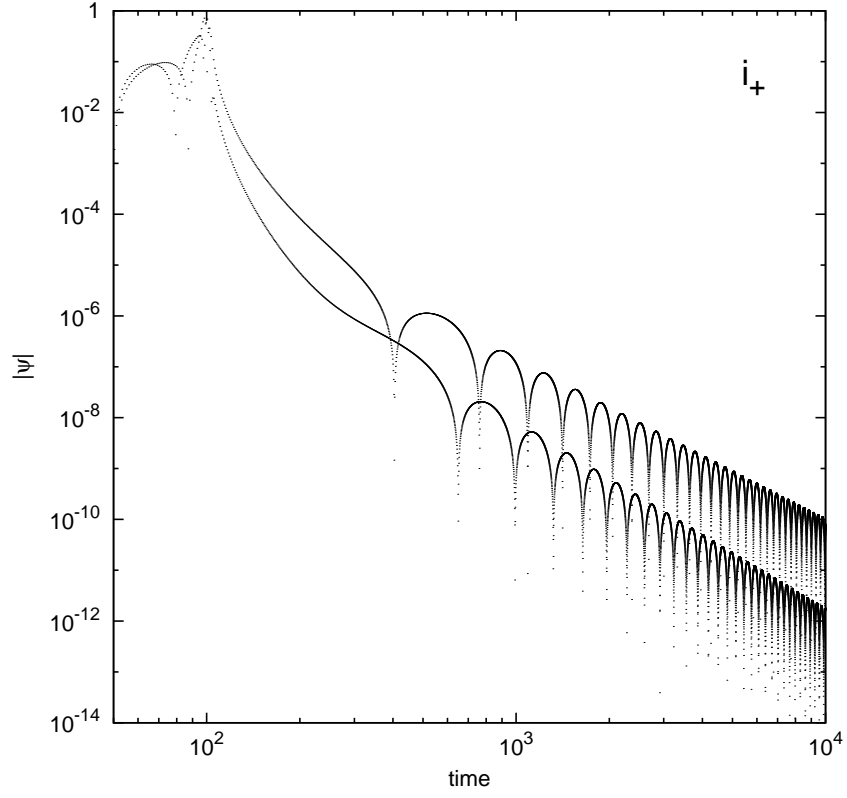[41] R.Moderski and M.Rogatko, Phys. Rev. D **60**, 104040 (1999).

FIG. 1: Evolution of the scalar field $| \psi |$ on the future timelike infinity i_+ as a function of t for different spacetime dimensions $d = 5, 6$. Calculation parameters: $m = 0.01$, $k = 0$, $n = 1$, $B = 0.9$. The power-law exponents are: -3.14 and -3.64 for $d = 5$, 6, respectively. The period of oscillation is $T = \frac{\pi}{m} \simeq 314.5 \pm 0.5$ for both curves.
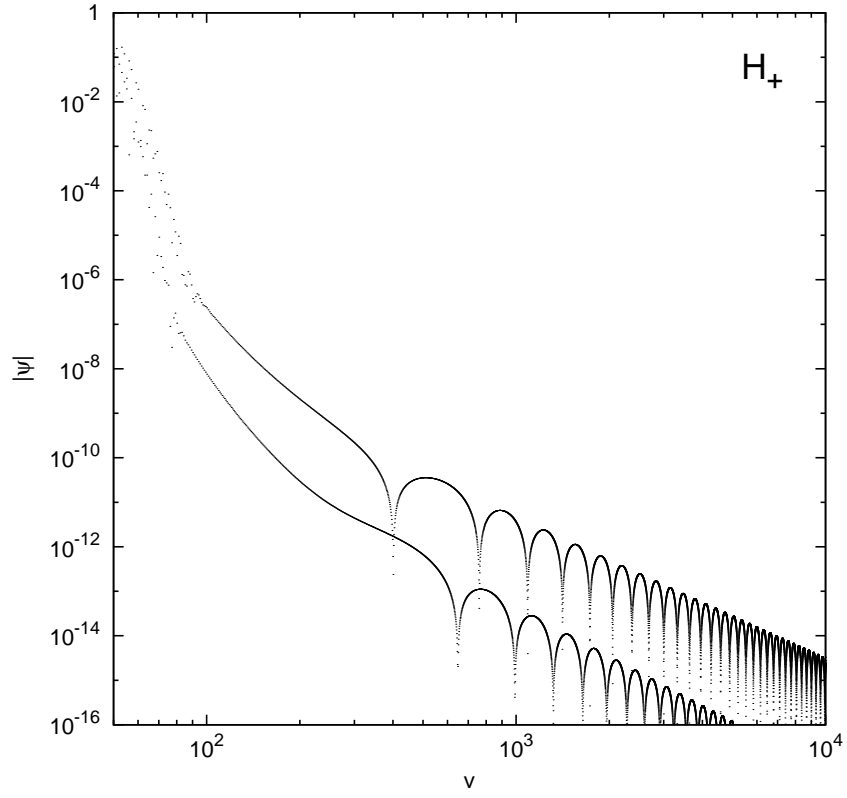
FIG. 2: Evolution of the scalar field $| \psi |$ on the black hole future event horizon H_+ (approximated by $\psi(u = 10^4, \ v)$) as a function of v for different spacetime dimensions $d = 5$, 6, upper and lower curve, respectively. Calculation parameters: $m = 0.01$, $k = 0$, $n = 1$, $B = 0.9$. The power-law exponents are: -3.15 and -3.67, for the worst bottom curve. The period of oscillation is $T = \frac{\pi}{m} \simeq 314.5 \pm 0.5$ to within 1% for both curves.
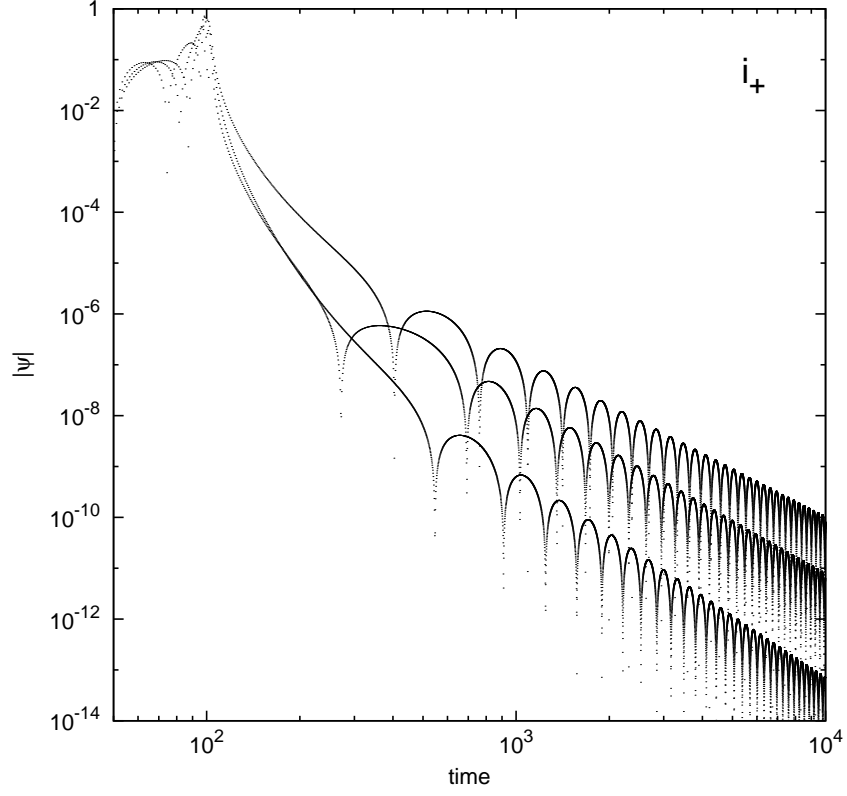
FIG. 3: Evolution of the scalar field $|\psi|$ on the future timelike infinity i_+ as a function of t for different $B = 0.9, 0.7, 0.5$ (curves from the top, respectively) and $d = 5$. Other calculation parameters: $m = 0.01$, $k = 0$, $n = 1$. The power-law exponents are: -3.14, -3.47 and -4.04. The period of oscillation is $T = \frac{\pi}{m} \simeq 314.5 \pm 0.5$ to within 0.2% for all curves.
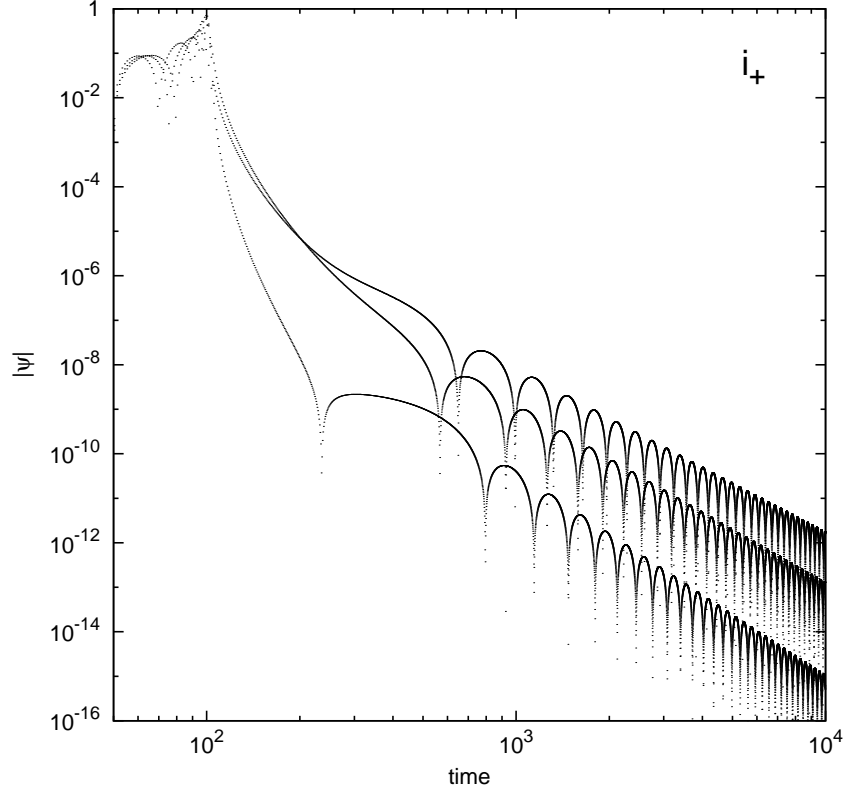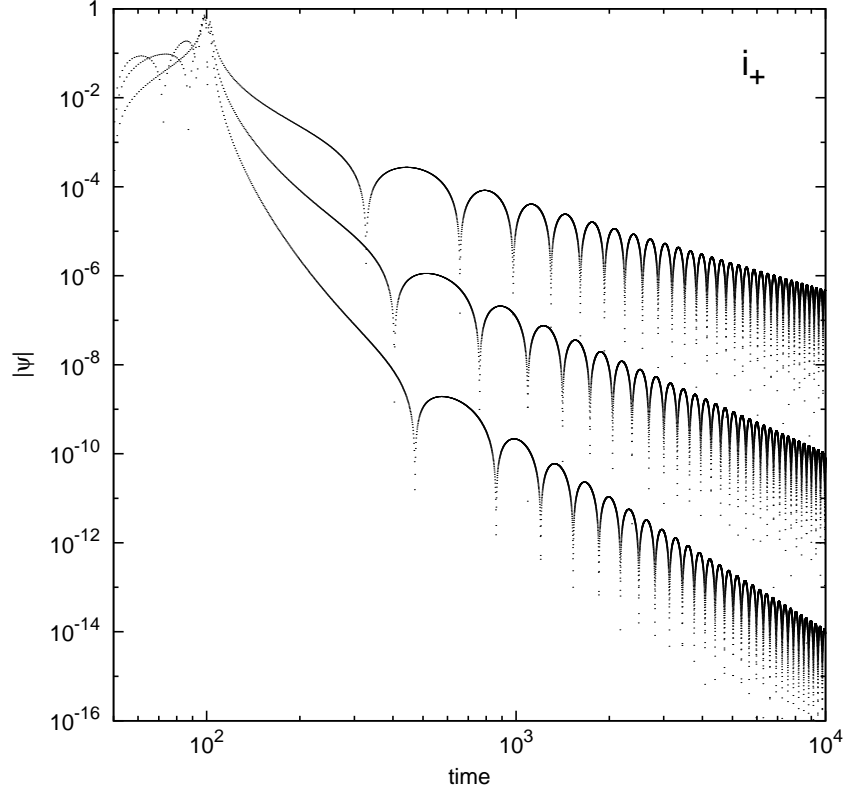
FIG. 4: Evolution of the scalar field $| \psi |$ on the future timelike infinity i_+ as a function of t for different $B = 0.9, \ 0.7, \ 0.5$ (curves from the top, respectively) in $d = 6$ spacetime. Other calculation parameters: $m = 0.01, \ k = 0, \ n = 1$. The power-law exponents are: $-3.64, \ -3.97$ and -4.54. The period of oscillation is $T = \frac{\pi}{m} \simeq 314.5 \pm 0.5$ to within 0.4% for all curves.
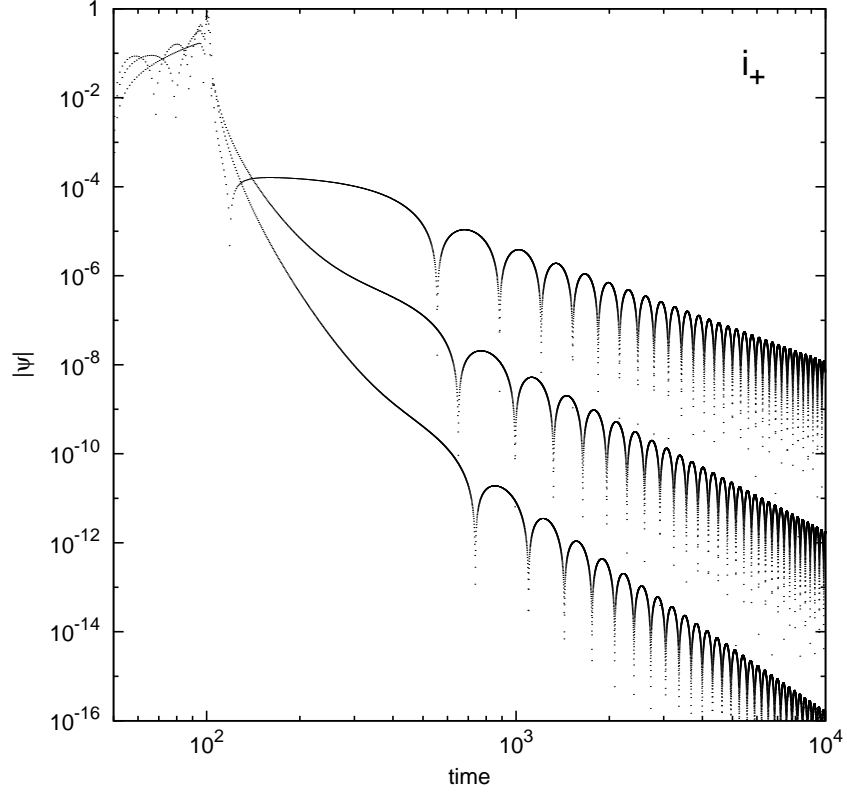
FIG. 5: Evolution of the scalar field $|\psi|$ on the future timelike infinity i_+ as a function of t for different $n = 0,\ 1,\ 2$ (curves from the top, respectively) in $d = 5$ spacetime. Other calculation parameters are: $m = 0.01$, $k = 0$, $n = 1$. The power-law exponents are: $-2.03,\ -3.14$ and -4.26. The period of oscillation is $T = \frac{\pi}{m} \simeq 314.5 \pm 0.5$ to within 0.3% for all curves.

FIG. 6: Evolution of the scalar field $| \psi |$ on the future timelike infinity i_+ as a function of t for different $n = 0, \ 1, \ 2$ (curves from the top, respectively) in $d = 6$ spacetime. Other calculation parameters in Fig.5. The power-law exponents are: $-2.53, \ -3.64$ and -4.78. The period of oscillation is $T = \frac{\pi}{m} \simeq 314.5 \pm 0.5$ to within 0.6% for all curves.
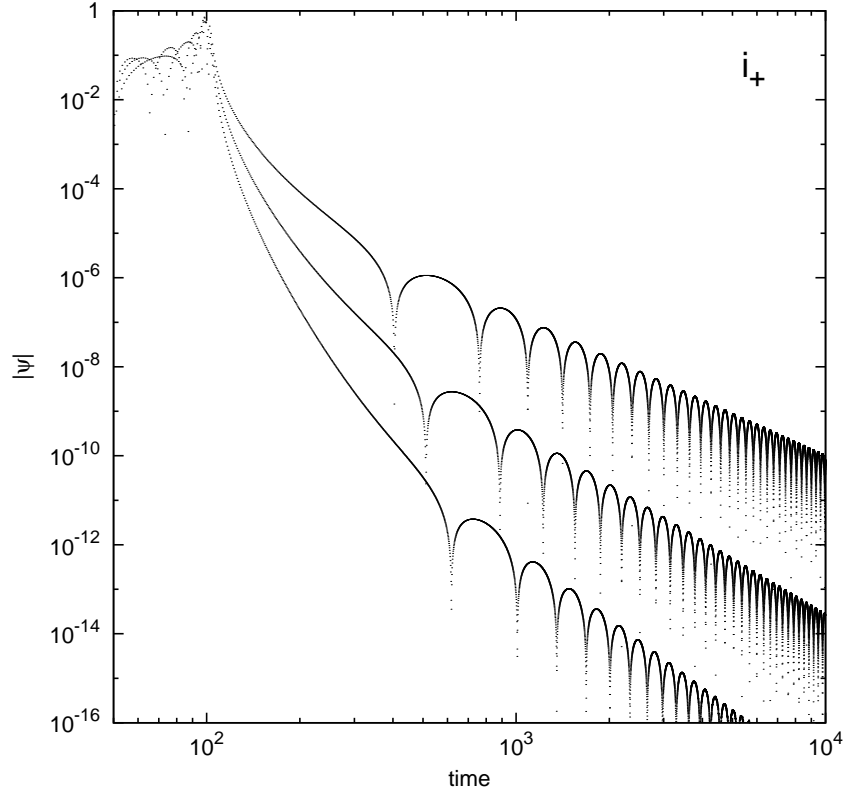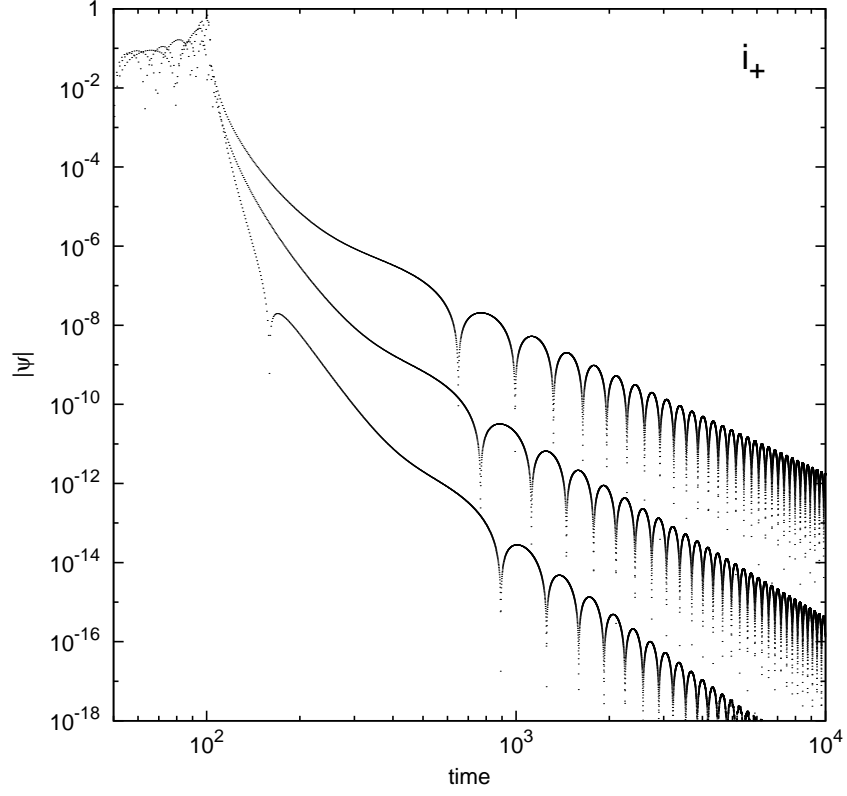
FIG. 7: Evolution of the scalar field $|\psi|$ on the future timelike infinity i_+ as a function of t for different $k = 0,\ 1,\ 2$ (curves from the top, respectively) in $d = 5$ spacetime. Other calculation parameters are: $m = 0.01$, $n = 1$, $B = 0.9$. The power-law exponents are: $-3.14,\ -4.15$ and -5.20. The period of oscillation is $T = \frac{\pi}{m} \simeq 314.5 \pm 0.5$ to within 1.7% for all curves.

FIG. 8: Evolution of the scalar field $| \psi |$ on the future timelike infinity i_+ as a function of t for different $k = 0, 1, 2$ (curves from the top, respectively) in $d = 6$ spacetime. Other calculation parameters in Fig.7. The power-law exponents are: -3.64, -4.67 and -5.60. The period of oscillation is $T = \frac{\pi}{m} \simeq 314.5 \pm 0.5$ to within 3.5% for all curves.
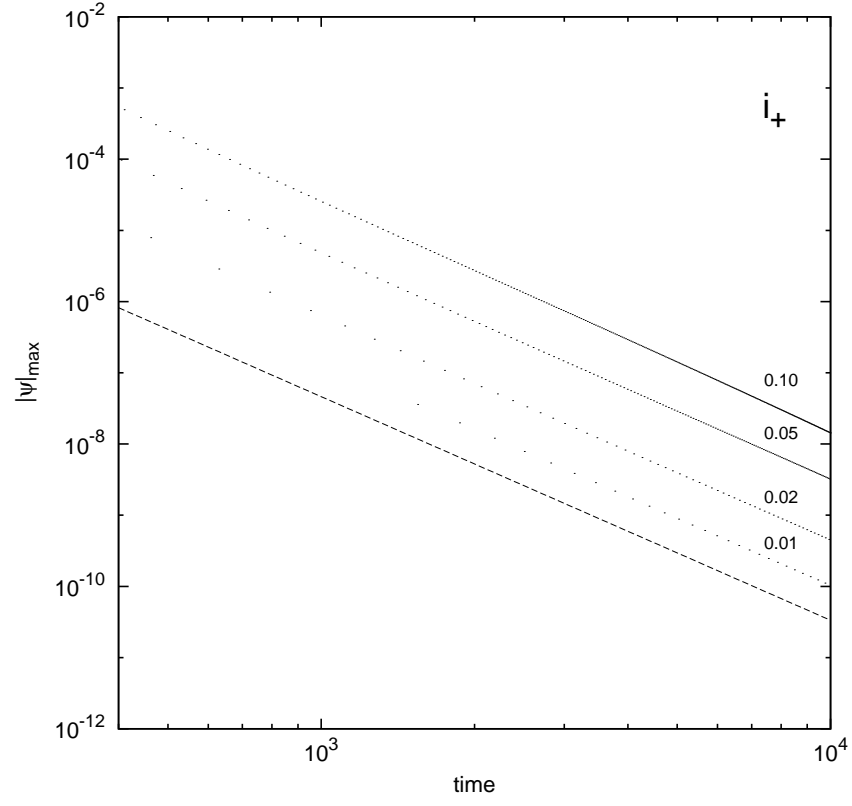
FIG. 9: Late-time decay rate of the field $| \psi |_{max}$ at i_+ for the five-dimensional case and different masses of the scalar field m (values written above the lines). Only maxima of the oscillations are showed. The dashed line has the slope equal to -3.14.
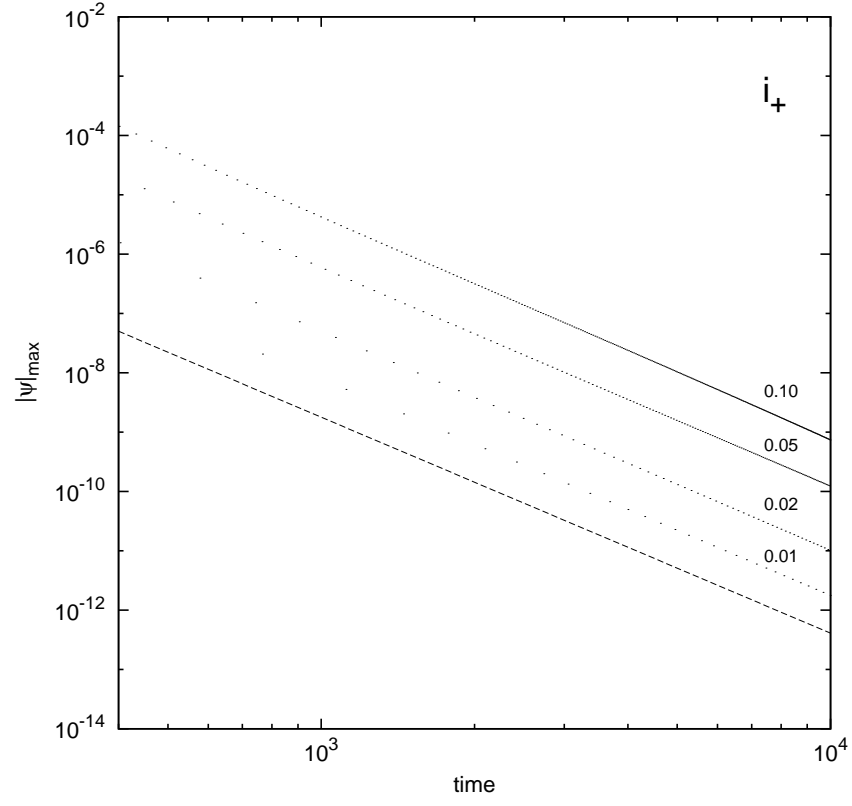
FIG. 10: Late-time decay rate of the field $| \psi |_{max}$ at i_+ for the six-dimensional case and different masses of the scalar field m (values written above the lines). Only maxima of the oscillations are showed. The dashed line has the slope equal to -3.64.